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Investigation of a Reinforced Concrete Highway Arch Bridge

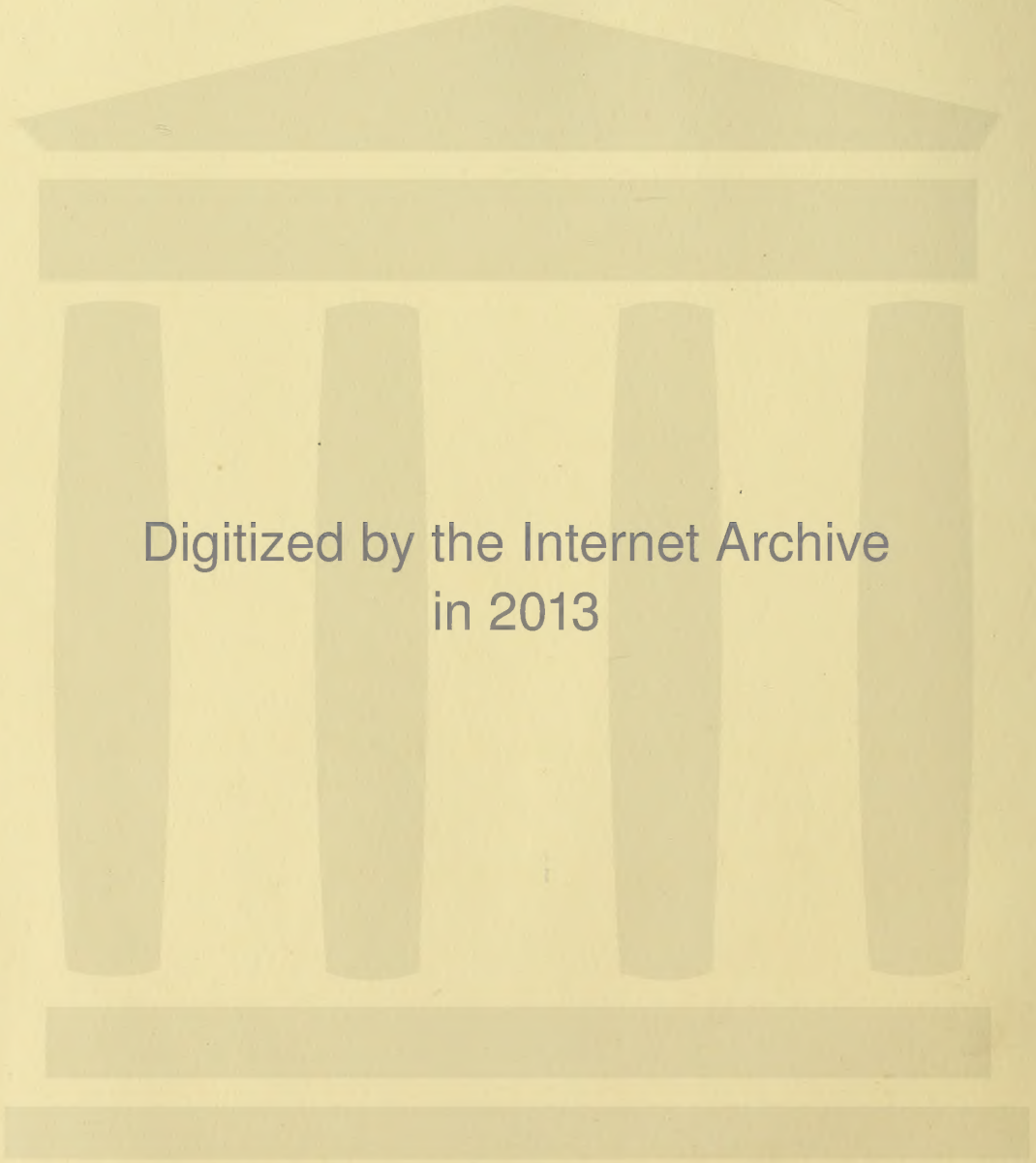
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INVESTIGATION
OF A
REINFORCED CONCRETE HIGHWAY ARCH BRIDGE

BY

DAVID MORRIS RIFF

THESIS

FOR THE

DEGREE OF BACHELOR OF SCIENCE

IN

CIVIL ENGINEERING

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May 25, 1914.

I hereby recommend that the thesis prepared under my direction by DAVID MORRIS RIFF entitled THE INVESTIGATION OF A REINFORCED CONCRETE HIGHWAY BRIDGE be accepted as fulfilling this part of the requirements for the degree of Bachelor of Science in Civil Engineering.

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Investigation of a Reinforced Concrete Highway Arch Bridge

Introduction

This thesis was undertaken by the writer for the purpose of acquainting himself with the methods of design, and with what is considered good practice in the construction of reinforced concrete structures. He has therefore selected as a specific subject, "The Investigation of a Reinforced Concrete Highway Arch Bridge over Island Creek, at Fourth Street in the City of Paducah, Kentucky."

The bridge is a reinforced concrete arch bridge designed for highway traffic. It consists of seven spans — a main arch span and six girder approach spans. The main span is a three-hinged reinforced concrete circular arch, having a length of 110 feet and a rise of 24 feet. The loads are transmitted to the arch rib

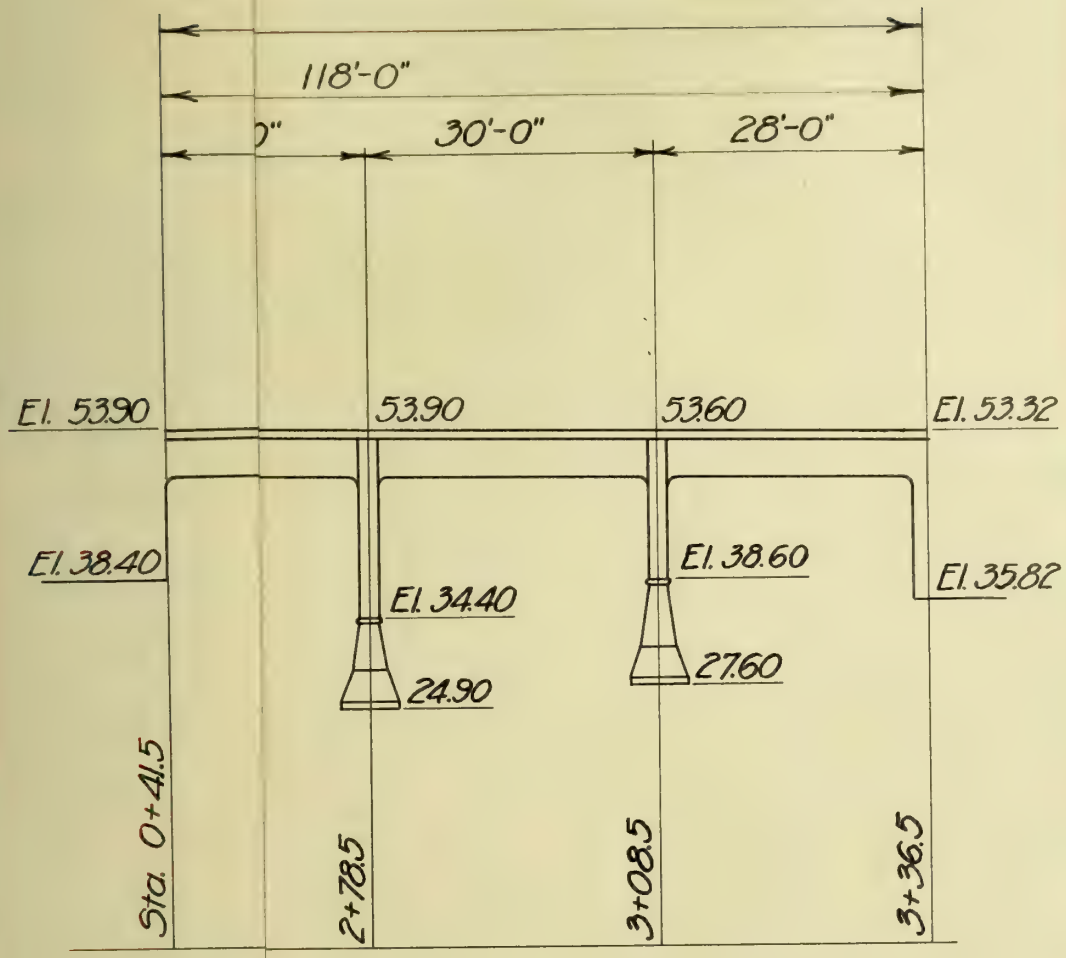
from the roadway above by reinforced concrete columns. The approach spans — two at the west end and four at the east end — are of the deck girder type. The floor system consists of reinforced concrete slabs carried on cross beams which transmit their loads to the longitudinal girders at the third points of the spans. The girders are supported on columns which rest on reinforced concrete pile foundations. The end spans are carried on reinforced concrete cantilever abutments, which are illustrated in Plate VI.

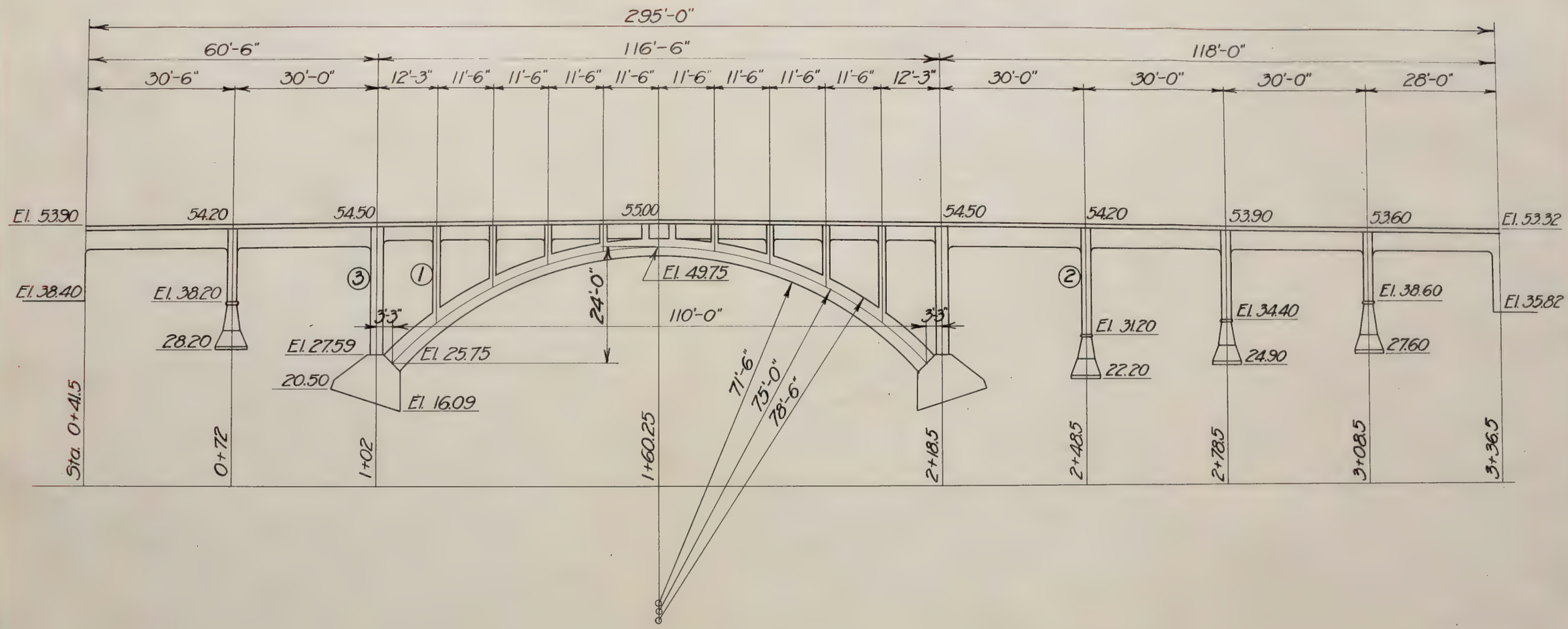
The bridge has a twenty-eight foot roadway with a six foot sidewalk on each side, the latter being carried on cantilever brackets. A concrete rail runs along the edge of each sidewalk and improves the general appearance of the bridge, which is illustrated by Plates I and II. Plate III shows the arrangement and dimensions of the various spans.

This investigation embodies the determination of the efficiency of the

various structural units of bridge, such as floor slabs, beams, girders, columns, footings, three-hinged arch, and abutments. In carrying on the investigation, the various parts of the work will be treated under the headings, and in the order indicated in the following outline:

1. Determination of Specifications.
2. Investigation of Floor Slabs.
3. Investigation of Floor Beams and Girders.
4. Investigation of Columns.
5. Investigation of Arch.
6. Investigation of Footings.
7. Investigation of Abutments.


118'-0"
0"
30'-0"
28'-0"
El. 53.90
53.90
53.60
El. 53.32
El. 38.40
El. 34.40
24.90
El. 38.60
27.60
El. 35.82
Sta. 0+41.5
2+78.5
3+08.5
3+36.5



LONGITUDINAL SECTION ON ℄ OF BRIDGE

PLATE III.

I. SPECIFICATIONS.[1]

Art. 1. Materials.

All concrete shall consist of cement, sand, and broken stone (the latter to be graded from $\frac{3}{8}$" to 1"), in the following proportions: 1 part cement, $2\frac{1}{2}$ parts sand, and 4 parts broken stone. Gravel may be substituted for the sand and stone, but wherever this is done, the gravel concrete must be of such proportions as to have the same density as that of the sand and broken stone concrete.

The reinforcing shall be plain round bars of medium steel.

---

[1] In writing these specifications, the following works were referred to: "General Specifications for Concrete Bridges," by Wilbur J. Watson; "Principles of Reinforced Concrete Construction," by Turneaure and Maurer; General Specifications for Bridge Work," - Illinois Highway Commission. (Ed. of Apr. 1913.)

Art. 2. Loads.

Dead Load: The dead load shall be considered as the total weight of the superstructure. The weight of reinforced concrete shall be taken as 150 lbs. per cu. ft. The weight of the wood block paving and sand cushion shall be taken as 50 lbs. per sq. ft. of surface.

Live Load: The following live loads shall be considered on the bridge in the investigation of the various parts of the structure: A concentrated load of 24 tons on two axles, ten feet apart, the rear axle carrying 16 tons on two wheels, spaced five feet apart; or a uniform live load of 125 lbs. per sq. ft. over the bridge floor. The live load on the sidewalk shall be considered as 100 lbs. per sq. foot.

Since the concrete floor of the bridge is covered with a 4" wood block pavement on a 2" sand cushion, and also since the concrete slab has a tendency to distribute a concentrated load, the two concentrated wheel loads will be considered equivalent to a uniform load distributed over a

rectangle 2 ft. in width perpendicular to the axle and 14 ft. in length parallel with the axis, making a uniform load in this rectangle of 32000 ÷ 28 or 1140 lbs. per sq. ft. for the rear axle, and 16000 ÷ 28 or 570 lbs. per sq. ft. for the front axle.

Art. 3. Unit Stresses.

The allowable working stresses in the concrete and steel shall be as follows:

Concrete

Direct Compression	500 #/□"
Compression due to transverse bending	700 #/□"
Diagonal Tension	40 #/□"
Bond stress	80 #/□"
Shear on reinforced concrete section	120 #/□"

Steel

Tension	16000 #/□"
Direct Compression	7500 #/□"

Foundations

The allowable pressure on the foundati

which is a hard clay, shall not exceed 6 tons per sq. ft. The bearing power of the piles will be calculated by the formula

$$P = \frac{2wh}{S + 0.1}$$

where P = safe load,
w = weight of hammer (steam)
h = fall of hammer in feet.
S = penetration under last blow (inches)

Art. 4. Assumptions for Design

(a) The modulus of elasticity of steel is taken as 30,000,000 lbs. per sq. in.; of concrete, 2,000,000 lbs. per sq. in. Therefore $n = 15$.

(b) The notation used will be the same as that in Turneaure and Maurer's "Principles of Reinforced Concrete Construction".

2. Investigation of Floor Slabs.

The investigation of the floor slabs will now be made. In this investigation, a strip one foot in width will be considered, and the stresses that are developed in it will be determined.

Art. 1. Roadway Floor Slab.

All of the roadway slabs have the same thickness and are reinforced alike; only the extreme case will be investigated therefore. This is the end slab of the arch span which is shown in Fig. 1, and for which

$L = 11'-6"$ c. to c. of bearings

depth $= 8"$

$d = 7"$

$A = .8836$ " (Reinforcing: $\frac{3}{4}"\phi$-6" o.c.)

$p = .0105$

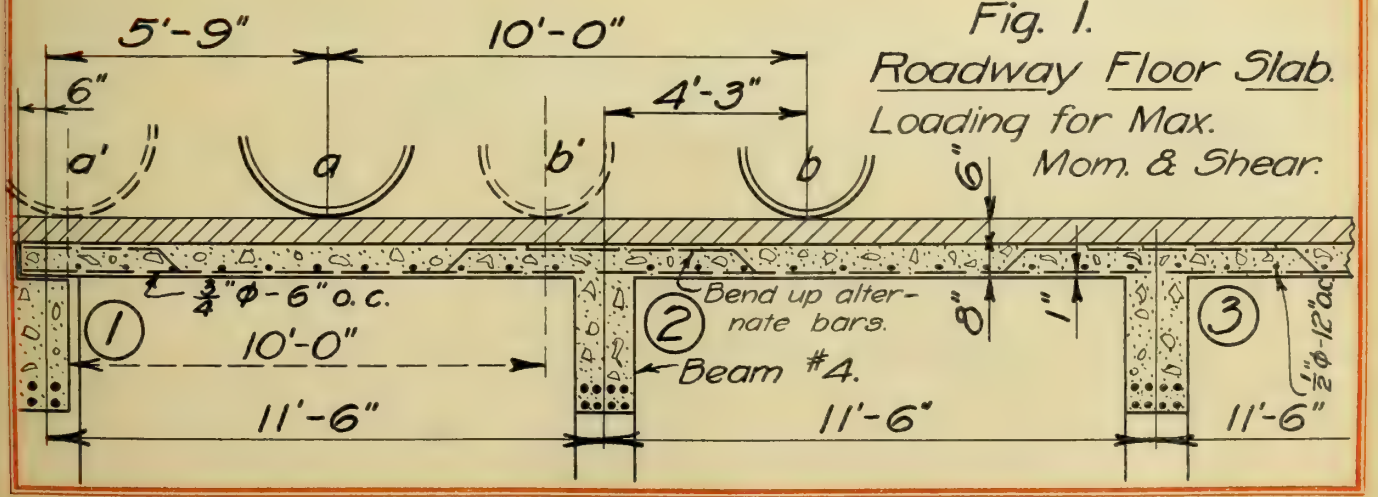


Fig. 1.
Roadway Floor Slab.
Loading for Max. Mom. & Shear.

Fig. 1 also shows the conditions of loading for maximum moment and maximum shear, the former being represented by the solid lines and the latter by the dotted lines.

D.L. per lin. ft.:

Wt. of Concrete = $\frac{8}{12} \times 150 = 100$

" " Pavement = $\underline{\quad 50 \quad}$

Total = 150 lbs.

L.L.: Wheel "a" = 1140 lbs. per ft. over two feet.
Wheel "b" = 570 lbs. per ft. over two feet.

Moments:

D.L. = $\frac{1}{10} \times 150 \times \overline{11.5}^2 = 1980$ #'

L.L.:

$$M_1 l + 4M_2 l + M_3 l = -P_1 l^2 (k - k^3) - P_2 l^2 (2k - 3k^2 + k^3) \quad (1)$$

$$M_2 l + 4M_3 l + M_4 l = -P_2 l^2 (k - k^3) \quad (2)$$

$$M_1 = M_4 = 0 \quad (3)$$

$$4M_2 + M_3 = -P_1 l (k - k^3) - P_2 l (2k - 3k^2 + k^3) \quad (1)$$

$$4M_2 + 16M_3 = -4 P_2 l (k - k^3) \quad (2)$$

$$-15M_3 = -P_1 l (k - k^3) - P_2 l (2k - 3k^2 + k^3) + 4P_2 l (k - k^3)$$

$$= \begin{cases} -2280 \times 11.5 \times (0.5 - 0.5^3) - 1140 \times 11.5 (2 \times .37 - 3 \times \overline{.37}^2 + \overline{.37}^3) \\ \quad + 4 \times 1140 \times 11.5 (.37 - \overline{.37}^3) \end{cases}$$

$$= -9840 - 5000 + 16750 = +1910$$

$$M_3 = -128 \text{ \#'}$$

$M_2 = -1140 \times 11.5 (.37 - .37^3) - 4(-128)$

$= -3678$ #'

$R_1 l - P_1(l - kl) = M_2$

$R_1 = \frac{-3678 + 2280(11.5 - .5 \times 11.5)}{11.5} = +820$ #

$Ma = 820 \times 5.75 = 4720$ #'

D.L. Mom. = 1980 #'

L.L. Mom. = 4720

Total = 6700 #'

$f_s = \frac{M}{pjbd^2} = \frac{6700 \times 12}{.0105 \times .86 \times 12 \times 7^2} = 15150$ #/□"

$f_c = \frac{M \times 2}{kjbd^2} = \frac{6700 \times 12 \times 2}{.43 \times .86 \times 12 \times 7^2} = 740$ #/□"

Efficiency-s = $\frac{16000}{15150} = 105.8\%$

Efficiency-c = $\frac{700}{740} = 94.6\%$

Shears:

D.L. = $150 \times .6 \times 11.5$ = 1040 #

L.L. = $P_1 + \frac{1}{4} P_2 (4 - 5k + k^3)$

$= 2280 + \frac{1}{4} \times 1140 \left[4 - \frac{5 \times 10}{11.5} + \left(\frac{10}{11.5}\right)^3\right] = 2380$

Total = 3420 #

Unit Shear = $3420 \div (7 \times 12) = 40.7$ #/□"

Efficiency-v = $\frac{40}{40.7} = 98.5\%$

Art. 2. Sidewalk Floor Slab.

The sidewalk slab which is to be investigated is the one adjacent to the roadway slab just considered. A section through the slab and the sidewalk beams is shown in Plate IV. The slab has the following dimensions and reinforcing:

$L = 6'-9"$

depth $= 4"$

$d = 3"$

$A = 0.22\ \square"$ (Reinforcing: $\frac{3}{8}"\phi - 6"$ o.c.)

$p = 0.00612.$

D.L. per lin. ft.:

Weight of concrete $= \frac{4}{12} \times 150 = 50\ \#$

Weight of cement finish $= \frac{1.5}{12} \times 125 = 15$

Total $= 65\ \#$

L.L. per lin. ft. $= 100\ \#$

Moments:

$$D.L. = \frac{1}{8} \times 65 \times \overline{6.75}^2 = 368\ \#'$$

$$L.L. = 600 \times \frac{3.75}{6.75} \times 3.38 - 100 \times \frac{\overline{3.38}^2}{2} = 560$$

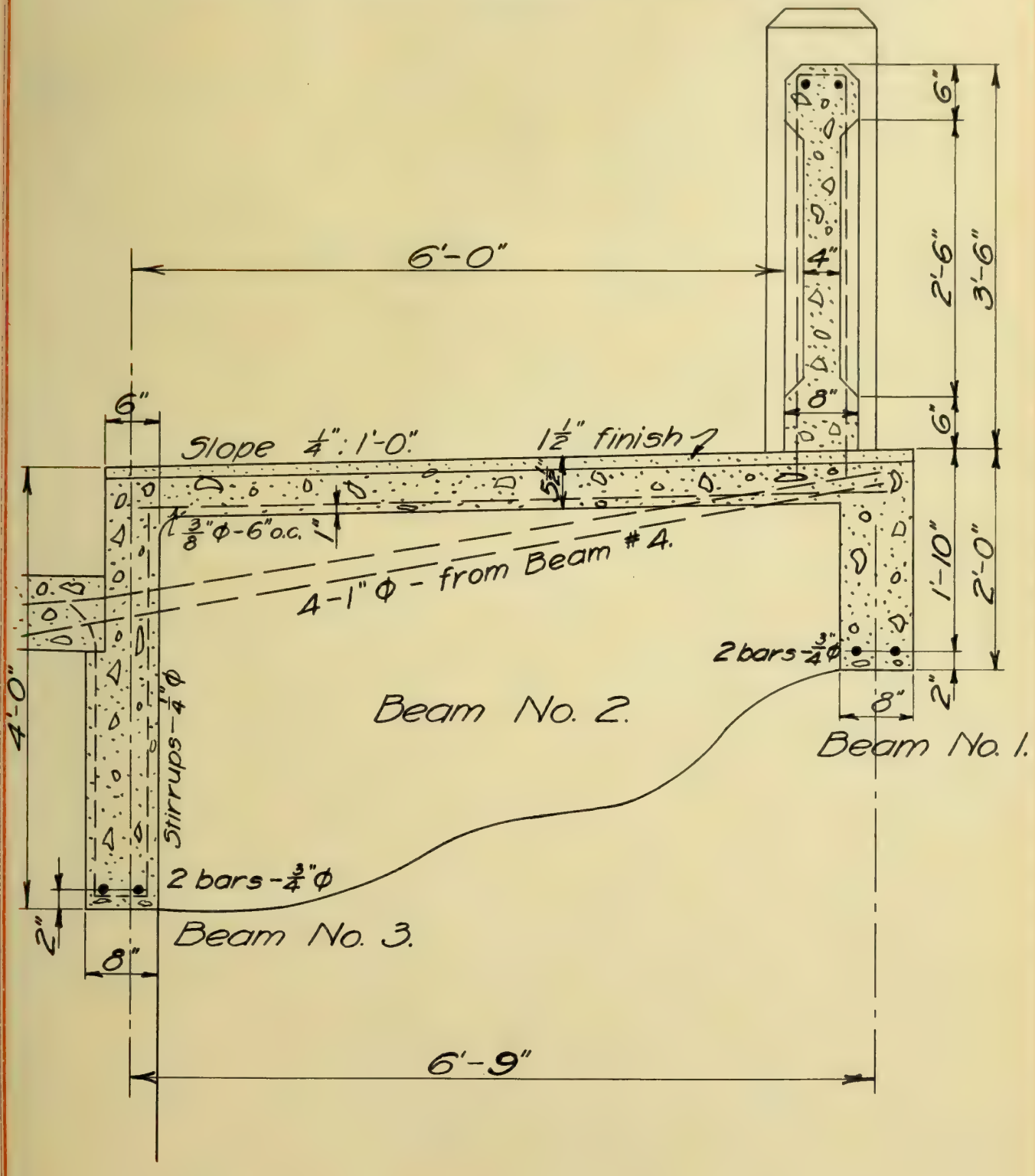
Total $= 928\ \#'$

$$f_s = \frac{928 \times 12}{0.00612 \times .87 \times 12 \times \overline{3}^2} = 19400\ \#/\square"$$

$$f_c = \frac{928 \times 12 \times 2}{.38 \times .87 \times 12 \times \overline{3}^2} = 625\ \#/\square"$$

PLATE IV

SECTION THRU SIDEWALK SLAB & BEAMS



$$\text{Efficiency}_{-s} = \frac{16000}{19400} = 82.5\ \%$$

$$\text{Efficiency}_{-c} = \frac{700}{625} = 112\ \%$$

Shears:

D.L. $= \frac{1}{2} \times 6.75 \times 65 = 220^{\#}$

Rail $= 250$

L.L. $= 600 \times \frac{3.75}{6.75} = 335$

Total $= 805^{\#}$

Unit Shear $= 805 \div (3 \times 12) = 22.2\ ^{\#}/\square''$

$$\text{Efficiency}_{-v} = \frac{40}{22.2} = 180\ \%$$

3. Investigation of Floor Beams & Girders.

As in the case of the floor slabs, only the critical beams and girders will be investigated, since it is the purpose of the writer to determine, not the efficiencies of all of the members or parts of the bridge, but rather the lowest efficiency of any set of members. The beams and girders will be investigated in the following order:

1. Sidewalk Beams.
2. Roadway Beams.
3. Girders.

Art. 1. Sidewalk Beams.

Beam #1.

This beam carries the end sidewalk slab of the arch span. It is shown in section on Plate IV.

$L = 11'-6"$

depth $= 2'-0"$

$d = 22"$

$b = 8"$

$A = 0.8836\ \square"$ (Reinforcing: $2-\frac{3}{4}"\phi$)

$p = .005$

D.L. per lin. ft.:

Wt. of Slab = $65 \times 3.42 = 220$ #

Concrete Rail = 250

Wt. of Beam = $.67 \times 2 \times 150 = 200$

Total = 670 #

L.L. per lin. ft. = $600 \times \frac{3.0}{6.75} = 267$ #

Moments:

D.L. = $\frac{1}{10} \times 670 \times \overline{11.5}^2 = 8850$ #'

L.L. = $\frac{1}{10} \times 267 \times \overline{11.5}^2 = 3530$

Total = 12380 #'

$$f_s = \frac{12380 \times 12}{.005 \times .89 \times 8 \times \overline{22}^2} = 9000 \text{ \#/□"}$$

$$f_c = \frac{12830 \times 12 \times 2}{.32 \times .89 \times 8 \times \overline{22}^2} = 280 \text{ \#/□"}$$

$$\text{Efficiency}_{-s} = \frac{16000}{9000} = 178\%$$

$$\text{Efficiency}_{-c} = \frac{700}{280} = 250\%$$

Shears:

D.L. = $.6 \times 670 \times 11.5 = 4620$ #

L.L. = $.6 \times 267 \times 11.5 = 1840$ #

Total = 6460 #

Unit Shear = $6460 \div (8 \times 22) = 37$ #/□"

$$\text{Efficiency}_{-v} = \frac{40}{37} = 108\%$$

Beam #2.

This is the cantilever bracket illustrated in Plate IV, which carries beam #1 and frames into the column.

$L = 7'-0''$

depth $= 4'-0''$ (at column)

$d = 33''$ $\quad p = .00795$

$b = 12''$

$A = 3.14\ \square''$ (Reinforcing: 4-1"ϕ)

Loads:

Concentrated Load at 6" from outer end:

D.L.: Wt. of Slab $= 65 \times 3.4 \times 11.5 = 2540^{\#}$

Wt. of Beam #1 $= 200 \times 11.5 = 2300$

Wt. of Rail $= 250 \times 11.5 = 2900$

Total $= 7740^{\#}$

L.L. $= 267 \times 11.5 = 3070^{\#}$

Uniform Load:

D.L. $= 3 \times 7 \times 150 = 3150^{\#}$

L.L. $= 0$

Moments:

D.L. $= \frac{3150 \times 7}{2} + 7740 \times 6.5 = 61300^{\#'}$

L.L. $= 3070 \times 6.5 = 20000$

Total $= 81300^{\#'}$

$$f_s = \frac{81300 \times 12}{.00795 \times .87 \times 12 \times 33^2} = 10800\ ^{\#}/\square''$$

$$f_c = \frac{81300 \times 12 \times 2}{.38 \times .87 \times 12 \times 33^2} = 455\ ^{\#}/\square''$$

$$\text{Efficiency}_{-s} = \frac{16000}{10800} = 148\%$$

$$\text{Efficiency}_{-c} = \frac{700}{455} = 154\%$$

Shears: (at support)

D.L. = 3150 + 7740 = 10890 #

L.L. = 3070

Total = 13960 #

Unit Shear = 13960 ÷ (12 × 33) = 35.3 #/□"

(to left of concentrated load)

Shear = $7740 + \frac{1}{14} \times 3150 = 7965$ #

Unit Shear = 7965 ÷ (12 × 21) = 31.6 #/□"

$$\text{Efficiency}_{-v} = \frac{40}{35.3} = 113\%$$

Beam #8.

Beam #8 is the cantilever bracket which carries the sidewalk beams of the approach spans; it corresponds to Beam #2 of the arch span.

L = 7'-0"
d = 33"
b = 12"
A = 4.72 □" (Reinforcing: 6 – 1" ϕ)
p = .0119

Loads:

P (6" from end of beam)

D.L.: Wt. of Slab = 65 × 3.4 × 10 = 2210 #
Wt. of Beam #1' = 200 × 10 = 2000
Wt. of Rail = 250 × 10 = 2500
Total = 6710 #

L.L. = 267 × 10 = 2670 #

W

D.L. = 3 × 7 × 150 = 3150 #
L.L. = 0

Following the same method of procedure as for Beam #2, the following stresses are found to be developed in the Beam:

f_s = 6500 #/□" Eff.$_s$ = 246 %
f_c = 350 #/□" Eff.$_c$ = 200 %
v = 31.7 #/□" Eff.$_v$ = 126 %

Beam #3.

Beam #3 supports the sidewalk slab. It is illustrated in Plate IV. The case under investigation is the same as for Beam #1 — the last beam of the arch span.

$L = 11'-6"$

depth $= 4'-0"$

$d = 34"$

$b = 8"$

$A = .8836\ \square"$ (Reinforcing: $2-\frac{3}{4}"\phi$)

$p = .00325$

D.L. per lin. ft.:

Wt. of Slab $= \frac{6.75}{2} \times 65 = 220\#$

Wt. of Beam $= 4 \times \frac{8}{12} \times 150 = 400\#$

L.L. $= 600 \times \frac{3.75}{6.75} = 334\#/.$ Total $= 620\#$

Moments:

$D.L. = \frac{1}{10} \times 620 \times \overline{11.5}^2 = 8180\#'$

$L.L. = \frac{1}{10} \left(600 \times \frac{3.75}{6.75}\right) \times \overline{11.5}^2 = 4420$

Total $= 12600\#'$

$$f_s = \frac{12600 \times 12}{.00325 \times .91 \times 8 \times \overline{34}^2} = 5540\ \#/\square"$$

$$f_c = \frac{12600 \times 12 \times 2}{.27 \times .91 \times 8 \times \overline{34}^2} = 133\ \#/\square"$$

$$\text{Efficiency}_{-s} = \frac{16000}{5540} = 289\%$$

$$\text{Efficiency}_{-c} = \frac{700}{133} = 526\%$$

Shears:

D.L. = .6 × 620 × 11.5 = 4275 #

L.L. = .6 × 334 × 11.5 = 2300

Total = 6575 #

Unit Shear = 6575 ÷ (8 × 34) = 24.2 #/□"

Web Reinforcement = $\frac{1}{4}$" ϕ bars – 12" o.c.

Value of $\frac{1}{4}$" ϕ stirrup = .049 × 2 × 16000

= 1570 #

= 1570 ÷ (12 × 8) = 16.5 #/□"

∴ Total Allow. Shear = 40 + 16.5 = 56.5 #/□"

Efficiency-$_V$ = $\frac{56.5}{24.2}$ = 233 %

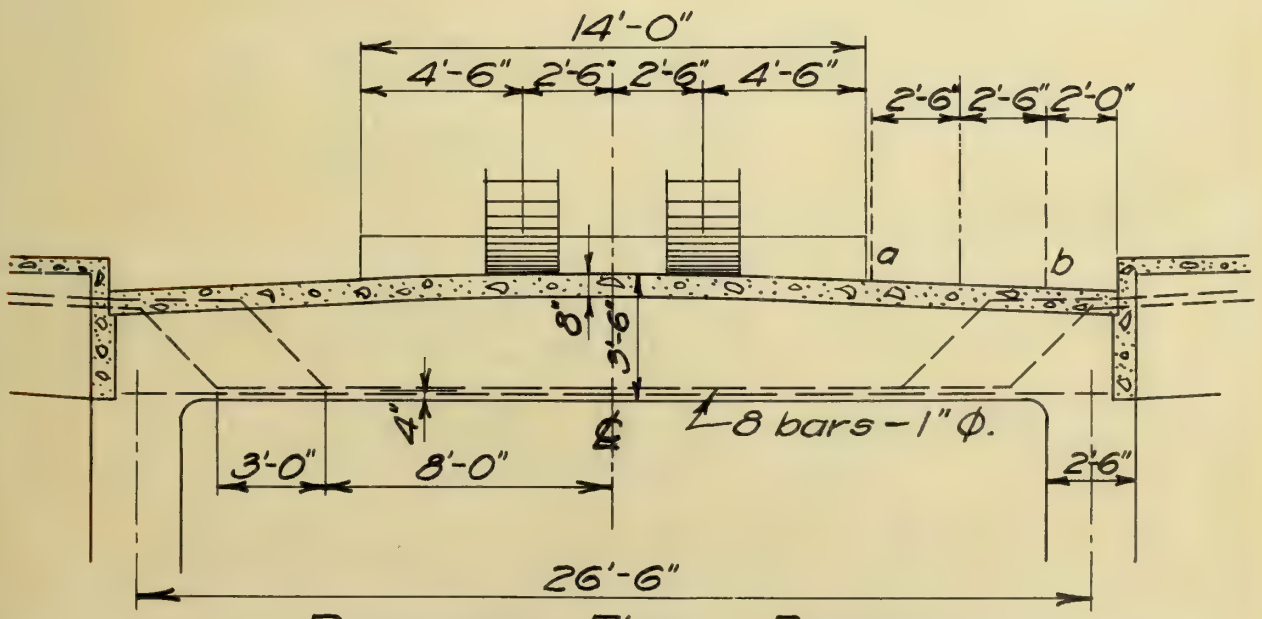
ART. 2. ROADWAY BEAMS.

Beam #4.

The floor system of this bridge, as has already been stated, consists of reinforced concrete floor slabs carried on cross beams. The beams, in the case of the arch span, transmit their loads directly to the columns, while, in the case of the girder spans, intermediate floor beams are provided which transmit their loads to the girders at the third points of the spans. These beams are all designed

alike, and since those of the arch span have the longer floor slabs to carry (see Plate III), one of them, Beam #4, will now be investigated. Fig. 2 shows the beam in elevation, together with the engine loadings for the conditions of maximum moment and shear. For both of these conditions the heavy wheels of the engine are directly above the beam and in the position shown in the figure. (The solid lines represent the loading for maximum moment; the dotted lines "a" and "b" represent the position of the centers of the wheels for maximum shear.)

Fig. 2.



Roadway Floor Beam
Loading for Maximum Moment & Shear.

$L = 26'-6"$

depth $= 3'-6"$

$d = 38"$

$b = 15"$

$A = 6.283\ \square"$ (Reinforcing: 8 bars – 1"ϕ)

$p = .011$

D.L. per lin. ft.:

Wt. of Floor Slab $= 11.5 \times 150 = 1725\ \#$

Wt. of Beam $= \frac{34 \times 15}{144} \times 150 = 532$

Total $= 2257\ \#$

L.L. = Engine loading as shown.

Moments:

D.L. $= \frac{1}{12} \times 2257 \times \overline{26.5}^2 =$ $132,000\ \#'$

L.L. = $166,000$

$W = \left[2280 + \frac{1140 \times 1.5}{11.5}\right] \times 14$

$= 34020\ \#$

$M = \frac{34020}{2} \times 13.25 - 2430 \times \frac{\overline{7}^2}{2}$

$= 166,000\ \#'$

Total Moment $= 298,000\ \#'$

$f_s = \frac{298,000 \times 12}{.011 \times .86 \times 15 \times \overline{38}^2} = 17500\ \#/\square"$

$f_c = \frac{298,000 \times 12 \times 2}{.43 \times .86 \times 15 \times \overline{38}^2} = 890\ \#/\square"$

Efficiency–s $= \frac{16000}{17500} = 91.5\%$

Efficiency–c $= \frac{700}{890} = 78.5\%$

Shears: (at end)

D.L. $= \frac{1}{2} \times 2257 \times 26.5 = 29{,}800^{\#}$

L.L. $= (32000 + 16000 \times \frac{1.5}{11.5}) \frac{22}{26.5} = 29{,}400$

Total $= 59{,}200^{\#}$

Unit Shear $= 59200 \div (32 \times 15) = 123^{\#}/\square''$

Eff. of section $= \frac{120}{123} = 97.5\%$

(at center)

D.L. $= 0$

L.L. $= (32000 + 16000 \times \frac{1.5}{11.5}) \frac{4.5}{26.5} = 5780^{\#}$

Total $= 5780^{\#}$

Unit Shear $= 5780 \div (38 \times 15) = 10^{\#}/\square''$

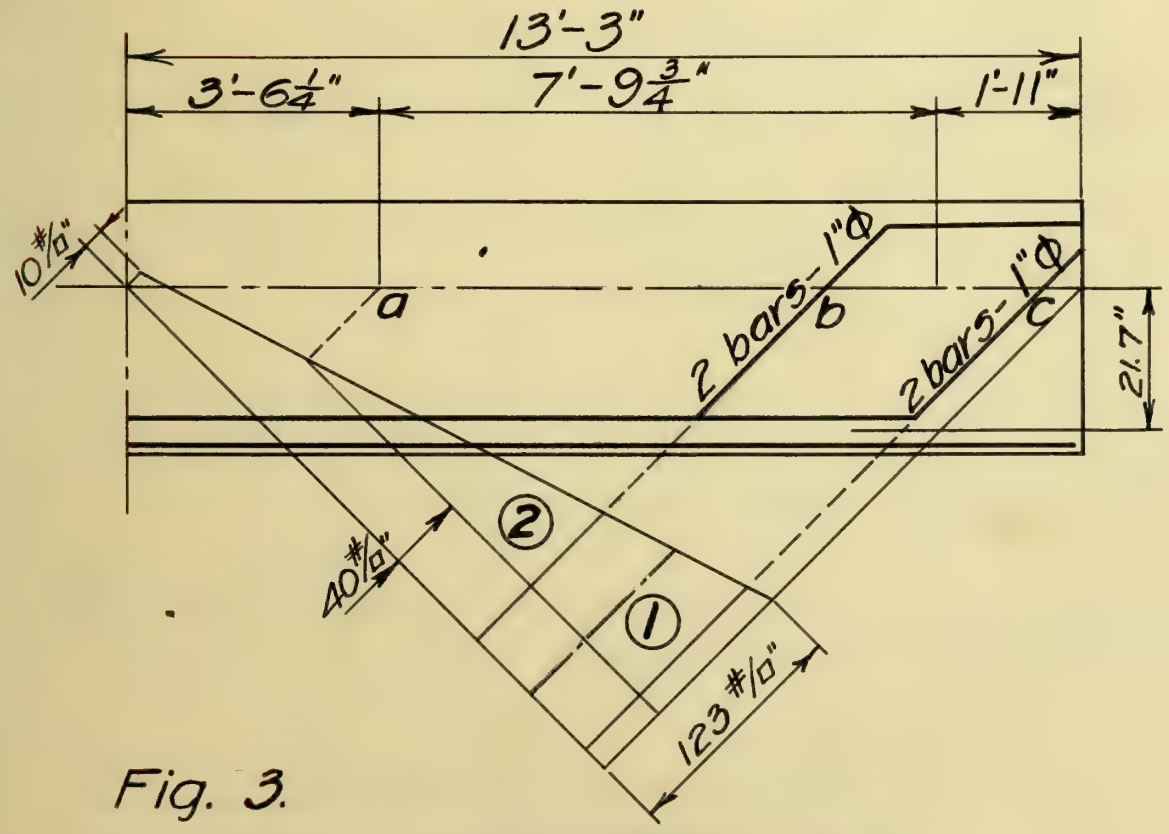


Fig. 3.

Sketch of Beam #4.
Illustrating Web Reinforcement.

Web Reinforcement:

Fig. 3 illustrates the web reinforcement of Beam #4. The point at which the shear is 40 #/□" is at a distance from the center of the beam equal to the value of "d" in the following:

$$\frac{13.25 - d}{13.25} = \frac{83}{113}$$

$$d = -\frac{83}{113} \times 13.25 + 13.25 = 3.52'$$

Section ②

Unit shearing stress $= 10 + (123 - 10)\frac{11.3}{13.25}$

$= 106.4$ #/□"

Carried by concrete $= 40.0$ #/□"

Shear to be carried by steel $= 66.4$ #/□'

Total shear to be carried by steel =

$$= \frac{66.4 \times 15 \times 7.81 \times 12}{2}$$

$= 46{,}700$ #

Value of 2-1"$\phi = \frac{16000 \times 1.5718}{1.41} = 17{,}900$ #

Efficiency of steel $= \frac{17900}{46{,}700} = 38.4\%$

Section ①

Unit shearing stress $= 10 + (123 - 10)\frac{12.3}{13.25}$

$= 115$ #/□"

Carried by concrete $= 40$

Shear to be carried by steel $= 75$ #/□

$= 75 \times 15 \times 23 = 25850$ #

Efficiency - steel $= 17900 \div 25850 = 69.5\%$

Fig. 3 shows that the web reinforcement is not only inadequate to carry the shearing stress, as has been found above, but also that the bent up bars are too far apart. The distance between bars should not exceed three-fourths the effective depth of the beam, which in this case equals .75 × 38" or 28.5"; the distances "ab" and "bc", however, are respectively 6'-3¾" and 3'-0".

Bond:

$$X = \frac{l}{\Sigma A} \times \Sigma a_1 + \cdots + a_n$$

where X = length of bar necessary before bending to develop strength of bar.

l = length of span

A = total steel area at center.

a_1, a_2, etc. = area of each rod up to the nth rod.

Section	No. of bars bent up	$\frac{l}{\Sigma A}$	$\Sigma a_1 + \cdots a_n$	X	Actual Length.
2	2	10.5	1.25	13.2'	16'-0"
1	4	10.5	1.78	18.75'	22'-0"

ART. 3. GIRDERS

The girder shown in Fig. 4 will now be investigated and its efficiency determined. The engine is in the middle of the span with its heavy wheels directly over cross-beam P_1 and its light wheels over P_2; the position of the engine on the roadway is the same as that shown for the condition of maximum shear in Beam #4. (see Fig. 2.)

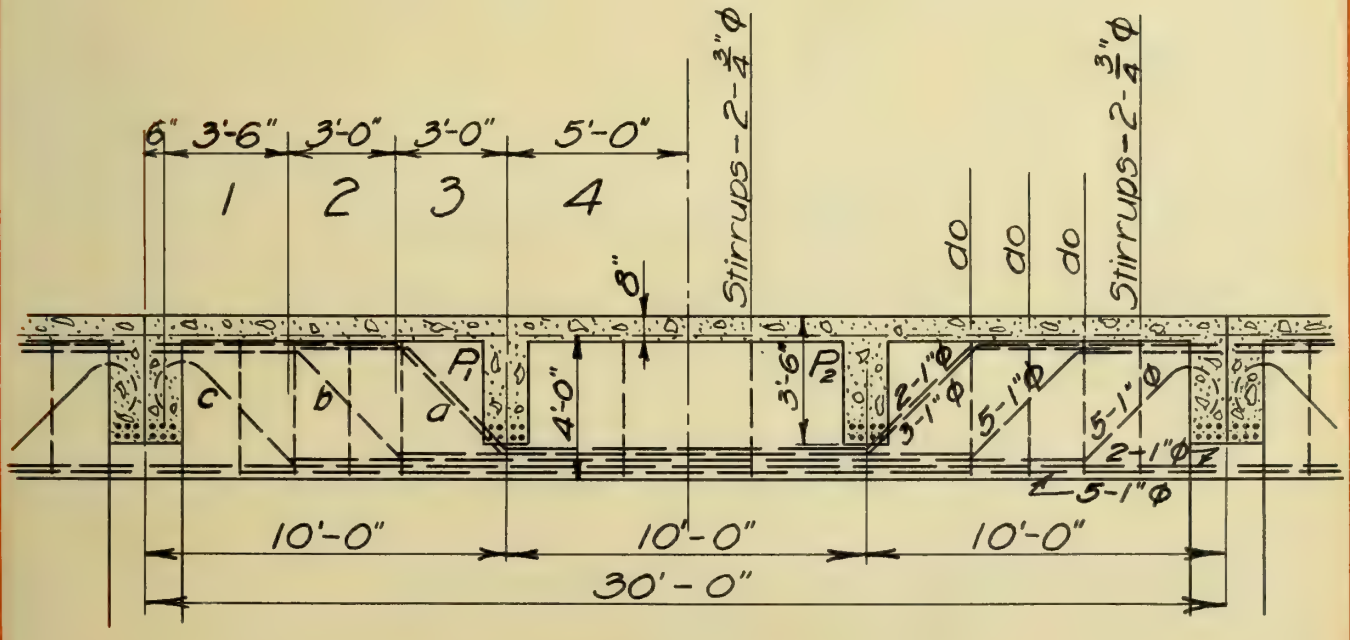


Fig. 4.

LONGITUDINAL SECTION OF GIRDER SPAN

$L = 30'-0"$

depth $= 4'-0"$

$d = 43"$

$b = 18"$

$A = 17.28^{\square"}$ (Reinforcing: 22-1"ϕ)

$p = .0223$

Loads:

P_1 D.L.:

Sidewalk Slab	$= 65 \times 3.4 \times 10 =$	$2210^{\#}$
Beam #1	$= 200 \times 10 =$	2000
Beam #2	$= 3 \times 7 \times 150 =$	3150
Rail	$= 250 \times 10 =$	2500
Roadway Slab	$= 150 \times 10 \times 13.5 =$	20250
Cross Beam	$= \frac{15 \times 34}{144} \times 150 \times 13.5 =$	7170
	Total	$= 37{,}280^{\#}$

L.L.:

Sidewalk Slab	$= 267 \times 10 =$	$2670^{\#}$
Engine	$= 32000 \times \frac{22.5}{27} =$	26650
	Total	$= 29{,}320^{\#}$

P_2 D.L. $= P_1$ D.L. $= 37280^{\#}$

L.L.:

Sidewalk Slab	$= 267 \times 10 =$	$2670^{\#}$
Engine	$= 16000 \times \frac{22.5}{27} =$	13325
	Total	$= 15{,}995^{\#}$

W D.L. per lin. ft.:

Sidewalk Slab = $65 \times 3.4 = 220^{\#}$

Girder = $4 \times 1.5 \times 150 = 900$

Total $= 1120^{\#}$

L.L. per lin. ft.:

Sidewalk = $334^{\#}$

R_1 D.L. = $37280 + 1120 \times 15 = 54080^{\#}$

L.L. = $29320 \times \frac{2}{3} + 15995 \times \frac{1}{3} + 334 \times 15 = 29880$

Total $= 83,960^{\#}$

Loads between R_1 and P_1 = $(1120 + 334) \times 10 = 14540^{\#}$

$P_1 = 29320 + 37280 = 66600$

Total $= 81140^{\#}$

Shear to right of $P_1 = 83960 - 81140 = 2820^{\#}$

Pt. of zero shear or max. mom = $2820 \div 1454$

(to right of P_1) $= 1.94'$

Moments:

D.L. = $54080 \times 11.94 - 37280 \times 1.94 - 1120 \times \frac{11.94^2}{2} = 493,800^{\#'}$

L.L. = $29880 \times 11.94 - 29320 \times 1.94 - 334 \times \frac{11.94^2}{2} = 275,900$

Total $= 769,700^{\#'}$

$$f_s = \frac{769,700 \times 12}{.0223 \times .83 \times 18 \times 43^2} = 15000 \ \#/\square''$$

$$f_c = \frac{769700 \times 12 \times 2}{.54 \times .83 \times 18 \times 43^2} = 1240 \ \#/\square''$$

$$\text{Efficiency-}_s = \frac{16000}{15000} = 107\%$$

$$\text{Efficiency-}_c = \frac{700}{1240} = 56.5\%$$

Shears: (at end)

D.L. = 54080 #

L.L. = 29880 #

Total = 83960 #

Unit Shear = $83960 \div (43 \times 18) = 108.5$ #/□"

(at center)

D.L. = 0

L.L. = $29320 \times \frac{2}{3} + 15995 \times \frac{1}{3} + 334 \times 15 \times \frac{3}{4} - 29320 - 334 \times 15$

$= -5700$ # = Total shear at center.

Unit Shear = $-5700 \div (43 \times 18) = -7.4$ #/□"

(to left of P_1)

Shear = $83960 - (1120 + 334)10 = 69420$ #

Web Reinforcement:

Section 1.

$V = (83960 - 14540 \times \frac{2.5}{10})42 = 3,380,000$ #

Bent bars = $\frac{5 \times .7854 \times 16000 \times .83 \times 43}{1.41} = 1,600,000$ #

Stirrups = $6 \times .4418 \times 16000 \times .83 \times 43 = 1,515,000$

Concrete = $43 \times 18 \times 40 \times 42 = 1,300,000$

Total value of section = 4,415,000 #

Efficiency = $4,415,000 \div 3,380,000 = 130\%$

Section 2.

$V = (83960 - 14540 \times \frac{5.5}{10})36 = 2,730,000$ #

Bent bars + Concrete = $1,600,000 + 1,300,000 \times \frac{6}{7}$

$= 2,710,000$ #

Stirrups = $\frac{4}{6} \times 1,515,000 = 1,010,000$

Total value of section = 3,720,000 #

Efficiency = $3,720,000 \div 2,730,000 = 136\%$

Section 3.

$V = (83960 - 14540 \times \frac{8.5}{10})\ 36 = 2{,}580{,}000^{\#}$

Bent bars + concrete $= 2{,}710{,}000^{\#}$

Stirrups $= \frac{1}{3} \times 1{,}515{,}000 = 505{,}000$

Total value of section $= 3{,}215{,}000^{\#}$

Efficiency $= 3{,}215{,}000 \div 2{,}580{,}000 = 125\%$

Section 4.

$V = \frac{2820 + 7.4}{2} \times 60 = 85{,}000^{\#}$

Stirrups $= \frac{4}{6} \times 1{,}515{,}000 = 1{,}010{,}000^{\#}$

Concrete $= 43 \times 18 \times 40 \times 60 = 1{,}860{,}000^{\#}$

Total value of section $= 2{,}870{,}000^{\#}$

Efficiency $= 2{,}870{,}000 \div 85{,}000 = 3370\%$

Length of horizontal bars:

$$X = \frac{l}{\sqrt{A}} \sqrt{a_1 + a_2 + \cdots a_n}$$

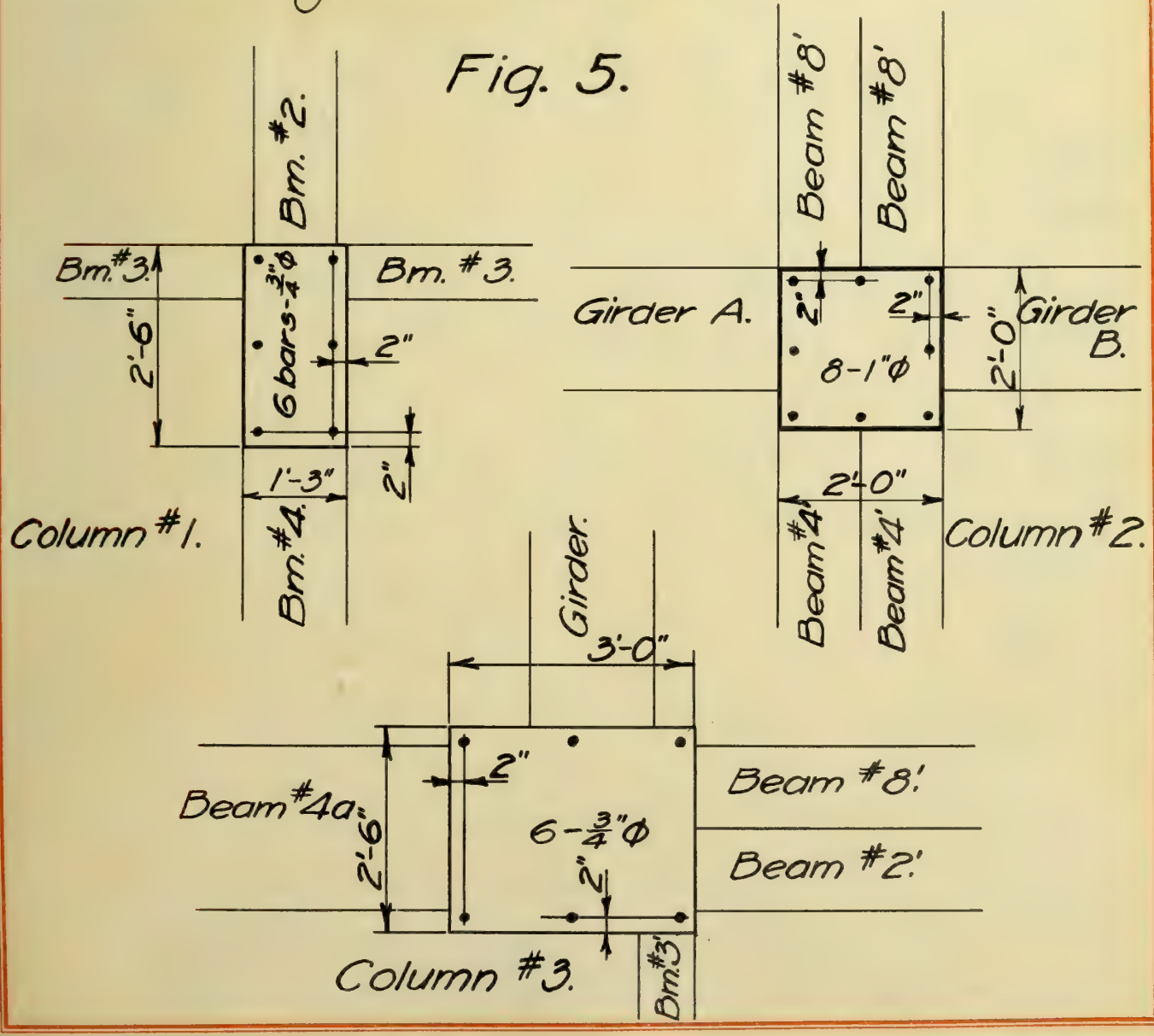
$$\frac{l}{\sqrt{A}} = \frac{30}{\sqrt{17.28}} = 7.2$$

Set	No. of Rods	$(a_1 + \cdots a_n)$	X	Actual Length	Eff.
a	5	3.93	14.3	10.0	70%
b	10	7.85	20.2	16.0	79%
c	15	11.78	24.7	22.0	89%
d	22	17.28	30.0	30.0	100%

4. Investigation of Columns.

Only three columns need be investigated to determine the efficiency of these structural units. A plan of each of the columns together with the beams framing into them is shown in Fig. 5. The columns are also marked by numbers on Plate III.

Fig. 5.



Column #1	Column #2	Column #3
$A = 11 \times 26 = 286^{\square''}$	$A = 20 \times 20 = 400^{\square''}$	$A = 26 \times 32 = 832^{\square''}$
$A_s = 6 \times .4418 = 2.65^{\square''}$	$A_s = 8 \times .7854 = 6.283^{\square''}$	$A_s = 6 \times .4418 = 2.65^{\square''}$
$p = .00927$	$p = .0157$	$p = .00318$

Loads:

Column #1—

Beam #2 = 13960#
Beam #3 = { 5490 / 5490
Beam #4 = 59400
Total = 84340#

Column #2—

D.L.: Girder A = 54080#
Girder B = 54080
Beam #4' = $20250 + \frac{24 \times 34}{144} \times 150 \times 13.5 =$ 31720
Beam #8' = $6710 + 2 \times 3150$ = 13010

L.L.: Girder A = $2670 + 334 \times 15$ = 7680
Girder B = $15995 \times \frac{2}{3} + 2670 \times \frac{1}{3} + 334 \times 15 =$ 16570
Beam 4' = 26650
Beam 8' = 2670
Total = 206,460#

In the above case, Column #2, the engine is on span B with the heavy wheels directly over Beam #4', in the same position as shown for maximum

shear in Fig. 2.

Column #3_

Place the engine on the girder span with its heavy wheels over Beam #4a, and in the same position on the roadway as for column #2.

D.L.: Girder = 54080#

Beam #3' = 3570

Beam #8' = $\frac{1}{2}$ × 6710 + 3150 = 6505

Beam #2' = $\frac{1}{2}$ × 7740 + 3150 = 7020

Beam #4a = $\frac{1150+1000}{2}$ = 1075

$2 \times 2.83 \times 150 \times 13.5$ = 11500

L.L.: Girder = 16570

Beam #3' = 1920

Beam #8' = $\frac{1}{2}$ × 2670 = 1335

Beam #2' = $\frac{1}{2}$ × 3070 = 1535

Beam #4a = 26650

Total = 131,760 #

$P' = A f_c [1 + (n-1) p]$

Column No.	P' lbs.	p	A sq.in	f_c lb./sq.in.	Efficiency %
1	84340	.00927	286	261	192
2	206,460	.0157	400	424	118
3	131,760	.00318	832	152	327

5. INVESTIGATION OF ARCH.

The three-hinged arch will now be investigated for the following three conditions of loading:

Case I: Live load over the entire arch.
Case II: Live load over the right half of arch.
Case III: Live load over middle half of arch – from b to b' (Plate V.)

Loads:

Columns A, B, and C: (similar to Col. #1.)

D.L.: Beam #2 = 10890#
Beam #3 = $\begin{cases} 3570 \\ 3570 \end{cases}$
Beam #4 = 22300
Total = 40330#

Wt. of Columns –

A = $\frac{15}{12} \times 2.5 \times 150 \times 14.25$ = 6680#
B = $6680 \times \frac{7.66}{14.25}$ = 3590#
C = $6680 \times \frac{3.15}{14.25}$ = 1475#

Wt. of Arch –

at A = 12.75 × 4.5 × 3.5 × 150 = 30100#
at B = 13.0 × 4 × 3.5 × 150 = 27300#
at C = 12.0 × 3.67 × 3.5 × 150 = 23100#

Total D.L.:

A	B	C
40330#	40330#	40330#
6680	3590	1475
30100	27300	23100
77110#	71220#	64905#

L.L. (A, B, and C):

Beam #2 = 3070#

Beam #3 = { 1920 / 1920

Beam #4 = $1440 \times 13.25 =$ 19100

Total = 26010#

Column D:

D.L.: Beam #2 = $7740 \times \frac{5.75+4.31}{11.5} + 3150 =$ 9920#

Beam #3 = 3570

Beam #3 = $3570 \times \frac{11.5-2.88}{11.5} =$ 2670

Beam #4 = $\left[\frac{11.5+8.62}{2} \times 100 + 532\right] \times 13.25 =$ 20400

36560

Wt. of Column = 125

Wt. of Arch = $10.2 \times 3.6 \times 3.5 \times 150$ = 19300

Total D.L. at D = 55985#

L.L.: Beam #2 = $3070 \times \frac{11.5-2.88}{11.5}$ = 2300#

Beam #3 = 1920

Beam #3 = $\frac{8.62}{11.5} \times 1920$ = 1440

Beam #4 = $\frac{5.75+4.31}{11.5} \times 1440 \times 13.25 =$ 16700

Total = 22360#

Column E:

D.L.: Beam #2 = $\frac{4.31+2.88}{11.5} \times 7740 + 3150 =$ 7980#

Beam #3 = $\frac{4.31+2.88}{5.75} \times 3570 =$ 4460

Beam #4 = [(4.31+2.88)100 +532]×13.25 = 16600

29040#

Wt. of Arch = 7×35×35×150 = 12900

Total D.L. at E = 41940#

L.L.: Beam #2 = 267×(4.31+2.88) = 1920#

Beam #3 = 334×(4.31+2.88) = 2400

Beam #4 = 125×(4.31+2.88) = 900

Total L.L. at E = 5220#

Table I, below, shows the dead load, live load, and total combined load at each of the points of the arch for all three cases. In Cases I and III, the loads are shown only for one half of the arch, since in these cases the arch is symmetrically loaded.

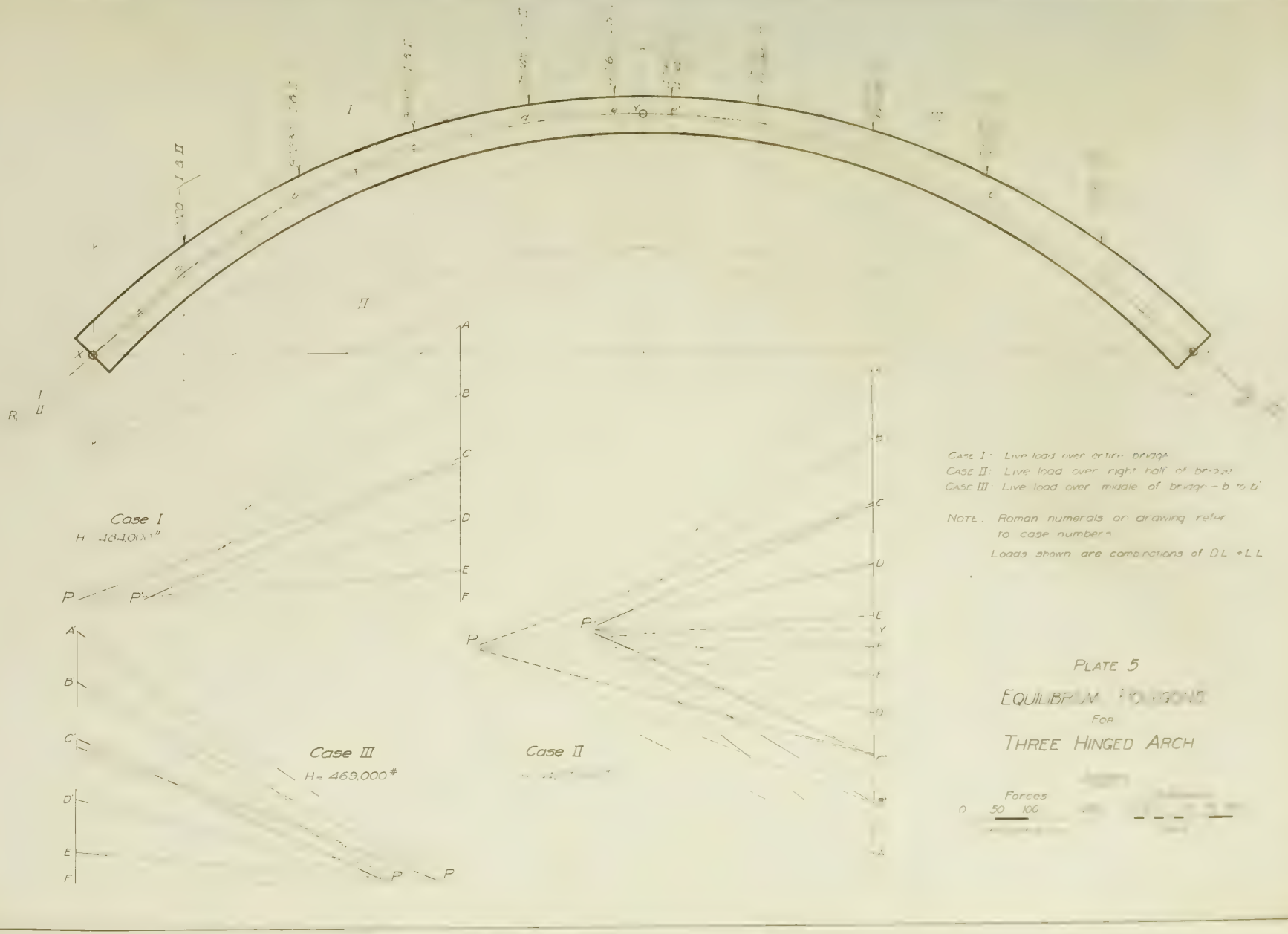
The arch was solved graphically for each of the three cases, the equilibrium polygons being shown on Plate 5. The reactions at the lower hinges act in the directions shown on Plate 5 and have the values given below:

TABLE I
Loads on Arch

Loading	Point	D. L.	L. L.	Total
CASE I	X	0	0	0
	a	77110	26010	103120
	b	71220	26010	97230
	c	64905	26010	90915
	d	55985	22360	78345
	e	41940	5220	47160
CASE II	X	0	0	0
	a	77110	26010	103120
	b	71220	26010	97230
	c	64905	26010	90915
	d	55985	22360	78345
	e	41940	5220	47160
	e'	41940	0	41940
	d'	55985	0	55985
	c'	64905	0	64905
	b'	71220	0	71220
	a'	77110	0	77110
	x'	0	0	0
CASE III	x'	0	0	0
	a'	77110	0	77110
	b'	71220	13005	84225
	c'	64905	26010	90915
	d'	55985	22360	78345
	e'	41940	5220	47160

	Case I	Case II	Case III
$R_1 =$	638,000#	578,000#	602,000#
$R_2 =$	638,000#	544,000#	602,000#

An examination of Plate 5 reveals the critical sections of the arch to be the second, third, and fourth


Case I
H = 184,000#
Case III
H = 469,000#
Case II
P
P'
A
B
C
D
E
F
A'
B'
C'
D'
E'
F'
Case I : Live load over entire bridge
Case II : Live load over right half of bridge
Case III : Live load over middle of bridge - b to b'
Note. Roman numerals on drawing refer to case numbers
Loads shown are combinations of D.L. + L.L.
PLATE 5
EQUILIBRIUM POLYGONS
FOR
THREE HINGED ARCH
Forces
0 50 100

sections to the left of x' (the sections between the points a' and b'; b' and c'; and c' and d'; referred to below as I'J'; K'L'; and M'N' respectively). At these sections the equilibrium polygon for Case II falls a considerable distance outside the middle third of the arch section. The stresses in both the steel and the concrete will now be found for these points of the arch, and the efficiency of the arch under the critical condition of loading thus determined.

In the solution of the stresses the following formulae will be employed:

$$f_c = \frac{T}{bd\,[1+(n-1)p]} + \frac{M}{(0.19+10.5p'')bd^2}$$

$$f_s = \frac{T}{bd\,[1+(n-1)p]} \times n - \frac{M}{0.85\,p'\,b\,d^2}$$

$$V = N \div bd'$$

where R = resultant pressure at any section

T = component of R parallel to the tangent to the neutral axis at the section.

N = component of R parallel to the radius of the neutral axis at the section.

H = true pole distance.

b = breadth of any section of the arch = 3.5'

d = distance between centers of bars at the extrados and intrados

d' = effective depth of the arch ring

$\bar{ac}$ = intercept (vertical) between the neutral axis and the true equilibrium polygon.

f_s = unit stress in the steel reinforcing

f_c = unit stress in the concrete

v = unit shearing stress on a section.

p = ratio of steel area to total area of section

p' = steel ratio for tensile reinforcement

p'' = steel ratio for compressive reinforcement

M = moment due to eccentricity of R or T

$= H \times \bar{ac}$

TABLE II

Stresses on Sections

	Section I'J'	Section K'L'	Section M'N'
R	500,000	467,000	445,000
T	500,000	467,000	444,000
N	10,000	11,000	23,000
d	3.66'	3.35'	3.2'
d'	3.91'	3.60'	3.45'
$\bar{ac}$	1.60'	1.40'	1.0'
p	.0086	.0094	.0099
p'=p"	.00403	.0044	.0046
f_c (intrados)	+624 #/□"	+645 #/□"	+542 #/□"
f_s (extrados)	-23370 #/□"	-20920 #/□"	-14670 #/□"
Efficiency-c	80%	77.5%	92.3%
Efficiency-s	69%	76.5%	109 %

A cross section of the arch at I'J' is

shown in Fig. 6. The arch rib has the same breadth throughout but varies in depth from 3'-6" at the crown to 5'-0" at the skewback. (The values of d in the table are the distances between the centers of the upper and lower reinforcing bars.)

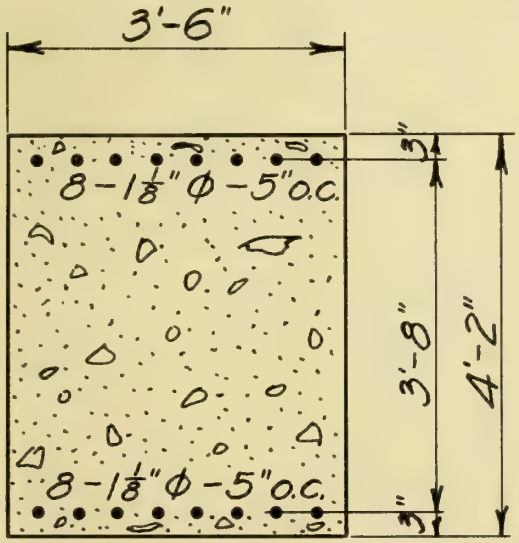


Fig. 6.

Section of Arch at I'J'.

6. Investigation of Footings.

In the investigation of the footings, only two of the footings need be considered, these being the footing which carries column no. 2 and the footing which carries column no. 3 and also acts as an abutment for the arch. These footings are shown in elevation on Plate III. The forms of the footings are such as to make it hardly worth while to investigate their strength, and, therefore, only the pressures on the foundations will here be considered. The writer has been furnished, by the engineer on the bridge, with a copy of the record of the driving of the piles, together with the safe loads on the piles, computed according to the Engineering News formula (factor of safety = 12). The safe bearing power of each foundation is therefore readily determined, from which may be figured the efficiency of the footing.

Column Footing #2:

Length = 10'-0"
Width = 6'-6"
Area = 65.0 □'
No. of Piles = 6
Average Safe Load on one pile = 36400#

Safe bearing power of foundation = P_B

$P_B = 6 \times 36400 + 65 \times 6000 = 608{,}400$#

Loads:

on Column #2 =	206,460#
Wt. of column = 2×2×19×150 =	11,400
Wt of footing =	35,800
Total load on found. =	253,660#

Efficiency = 608,400 ÷ 253,660
= 240 %

Column Footing #3:

Base = 15'-0" × 14'-6" = 217.5 □'
No. of Piles = 16
Average Safe Load on one pile = 31,800#
P_B = Safe bearing power of foundation.
= 16 × 31800 + 217.5 × 6000 = 1,813,800#

Loads:

on Column #3 = 131,760#

Wt. of Column = 2.5x3x23.4x150 = 26,325

Wt. of Footing = 151,000

Total Vertical Load = 309,085#

Reaction of Arch = 638,000#

The resultant of the above loads on the foundation and its component normal to the base of the footing (N) are determined graphically on Plate VI

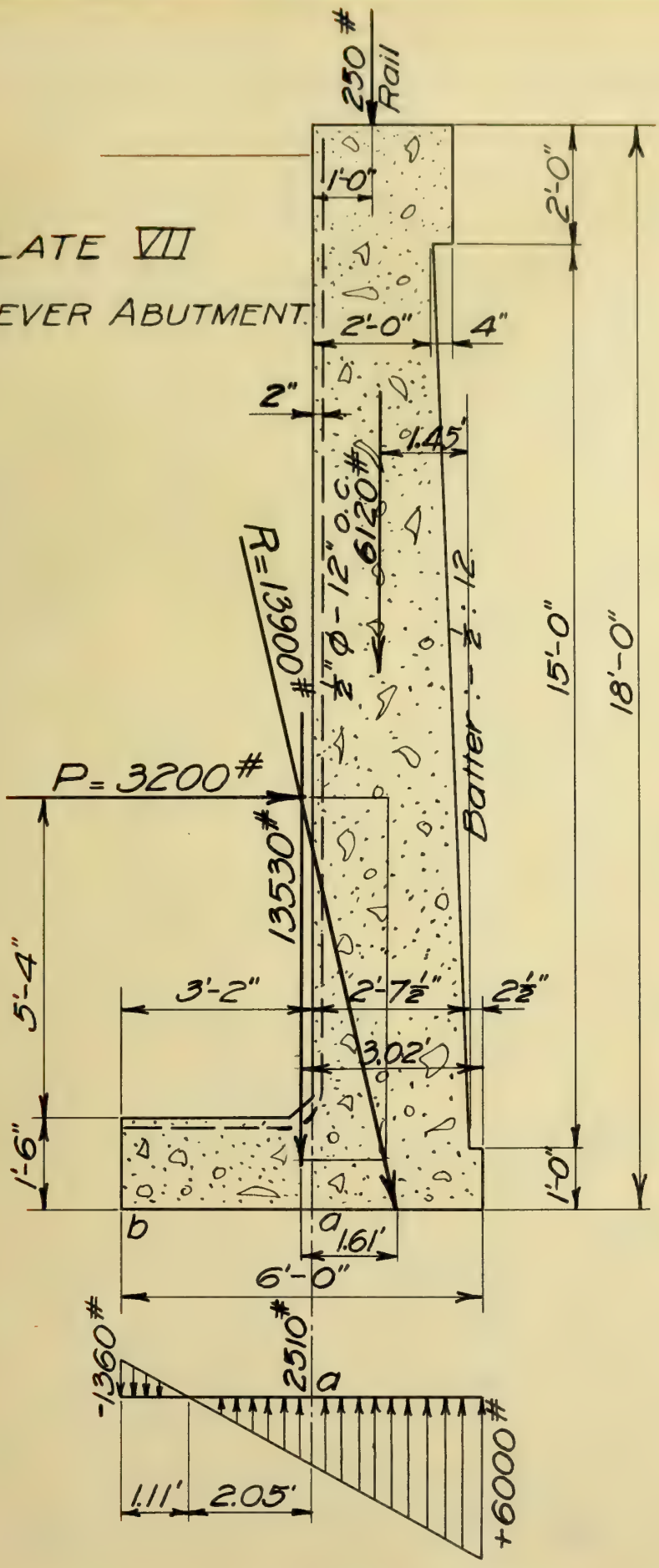
N = 835,000#

Efficiency = 1,813,800 ÷ 835000

= 217 %.

PLATE VII

CANTILEVER ABUTMENT.



7. INVESTIGATION OF ABUTMENTS.

The abutments of this bridge are of the reinforced concrete cantilever type. Plate VII shows the abutment for which the stresses are determined below. The condition assumed for this investigation is: the backfill is already in place but the floor slab above is as yet not constructed. The base of the abutment, or the footing, is approximately .45 of its height, and the equivalent fluid pressure assumed for the earth behind the wall is 25 lbs. per cu. ft.

Earth Pressure:

$$P = \tfrac{1}{2} p h^2 = \tfrac{1}{2} \times 25 \times \overline{16}^2 = 3200^{\#}$$

$$x = 16 \div 3 = 5.33'$$

Resisting Pressures:

$$\text{Wt. of Wall} = \left(2.33 \times 2 + \frac{2+2.6}{2} \times 15\right) \times 150$$

$$= 5870^{\#}$$

$$\text{c.g.} = \frac{\left[4.66 \times 1.46 + 2 \times 15 \times 1.6 + \frac{.6 \times 15}{2} \times .4\right] \times 150}{5870}$$

$$= 1.44' \quad \text{from toe of wall.}$$

Wt. of Rail $= 250^{\#}$ c.g. $= 1.6'$ from toe

Total Resisting Pressure $= 5870 + 250 = 6120^{\#}$

Distance from toe $= 1.44 + \frac{250}{5870} \times 1.6 = 1.45'$

Moments:

Overturning = $3200 \times 5.33 = 17060^{\#'}$

Resisting = $6120 \times 1.45 = 8880$

Resultant = $8180^{\#'}$

Moment of Steel = $.1963 \times 16000 \times 2.46$

$= 7730^{\#'}$

Efficiency = $7730 \div 8180 = 94.5\%$

Footing.

Earth Pressure = $3200^{\#}$

Resisting Pressure:

Wt. of Footing = $6 \times 1.5 \times 150 = 1350^{\#}$

" of Earth = $3.16 \times 16 \times 120 = 6060$

" of Wall + Rail = 6120

Total = $13530^{\#}$

Point of application = $[1350 \times 3 + 6060 \times 4.41 + 6120 \times 1.66] \div 13530$

$= 3.02'$ from toe.

Resultant Pressure = $\sqrt{13530^2 + 3200^2} = 13900^{\#}$

$r = \frac{3200}{13530} \times (5.33 + 1.5) = 1.61'$

Factor of safety (overturning) = F

$F = \frac{3.02}{1.61} = 1.88$

Factor of Safety against Sliding:

Coefficient of friction = 0.5

$F = \frac{0.5 \times 13530}{3200} = 2.12$

Footing Design:

$$\text{Pressure on Foundation} = \frac{W}{l} \pm \frac{6Wd}{l^2}$$

$$= \frac{13900}{6} \pm \frac{6 \times 13900 \times 1.59}{6^2}$$

$$= 6000\ \#/\square' \text{ maximum}$$

$$= -1360\ \#/\square' \text{ minimum}$$

x = distance from pt. of 0 pressure to heel.

$$x = \frac{1360}{6000}(6 - x)$$

$$x = 1.11'$$

$$\text{Pressure at "a" (Plate VII)} = \frac{3.16 - 1.11}{6.0 - 1.11} \times 6000$$

$$= 2510\ \#$$

$$\text{Upward Pressure on "ab"} = \frac{2510 \times 2.05}{2} - \frac{1360 \times 1.11}{2}$$

$$= 1825\ \#$$

$$\text{Pt. of Application} = \frac{-2580 \times .683 + 755 \times 2.79}{1825} = .19' \text{ from "a"}$$

Downward Pressure of Earth and Footing:

Earth =		= 6060 #
Footing =	3.16 × 1.5 × 150 =	715 #
	Total	= 6775 #

$$\text{Moment}_{-a} = 6775 \times 1.58 - 1825 \times 0.19 = 10350\ \#'$$

$$R = \frac{M}{bd^2} = \frac{10350 \times 12}{12 \times 16^2} = 40.5$$

Assume $f_s = 16000\ \#/\square''$

$\therefore p = .0028$

$A_s = .0028 \times 16 \times 12 = .538\ \square''$

$$\text{Eff.}_s = \frac{.1963}{.538} = 36.5\%$$

$Shear_{-a} = 6775 - 1825 = 4950^{\#}$

Unit Shear $= 4950 \div (12 \times 16) = 25.8\ ^{\#}/\square''$

Efficiency $= 40 \div 25.8 = 155\%$

Conclusion.

The above investigation, in the opinion of the writer, shows this bridge to be one that is well designed, at least insofar as it is in accordance with theoretical considerations. True, in many cases the efficiency of the various units falls considerably below 100%, and in others the efficiency is very much above that mark, but the writer believes that the former, in a great part, is due to the fact that the bridge was designed for a live load of 100 #/□' over the bridge, or an 18-ton engine, which is much lower than the live load assumed in this investigation (see Spec., Art. 2, p. 5); and that the latter may be accounted for by the fact that in this bridge, as in many other structures, certain parts must be designed considerably larger than is actually necessary in order to maintain the proper proportion between the units.

The writer's limited practical

experience in reinforced concrete construction makes it impossible for him to make any authoritative statement as to the economy or other practical considerations of the design. However, he is of the opinion that the bridge combines a theoretical and practical design with a general neat appearance.

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